Applied Maths II Final Exam

ADAMA SCIENCE AND TECHNOLOGY UNIVERSITY
SCHOOL OF NATURAL SCIENCE
DEPARTMENT OF MATHEMATICS
Final EXAM FOR APPLIED MATHS II (MATH 132)

PROGRAM: REGULAR
DATE: Monday Jan28, 2013
TIME ALLOWED: 3hrs

NAME_
ID NO.
DEPARTMENT__
YEAR

GENERAL INSTRUCTION:

THIS EXAM HAS TWO PARTS: SHORT ANSWER PART CONTAINS TEN QUESTIONS WHICH WORTH 20 POINTS AND WORKOUT PART CONTAINS FIVE QUESTIONS, WHICH WORTH 20 POINTS. ATTEMPT EACH QUESTIONS ON THE SPACE PROVIDED ACCORDING TO THE INSTRUCTIONS GIVEN.

FOR INSTRUCTOR USE ONLY

SHORT ANSWER	1	2	3	4	TOTAL

PART I: GIVE THE SHORT AND MOST SIMPLIFIED ANSWER

(Each question worth 2 pts)

1. Let $f(x,y) = e^{\sqrt{1-x^2-y^2}}$, then
   a. Domain of $f(x,y)$ = $1-x^2-y^2 \geq 0$
   b. Range of $f(x,y)$ = $0 \leq x^2+y^2 \leq 1$
2. Find the limit (if exist)
   a. $\lim_{(x,y)\to(0,0)}\left(\frac{x^2y^2}{x^3-y^3}\right) =$ 0
   b. $\lim_{(x,y)\to(1,1)}\left(\frac{x^2-xy}{\sqrt{x}-\sqrt{y}}\right) =$ 3
3. Find a function $g(x)$ such that the function $f(x,y)$ is continuous for

$$f(x,y) = \begin{cases} (1+xy)^{\frac{2}{y}}, & if\ y \neq 0 \\ g(x) & if\ y = 0 \end{cases}$$

   ; $g(x) =$ e^{2x}

4. Let $f(x,y,z) = e^{xy}\sin(z)$,then
   a. $f_{zyx}(1,1,0) =$ 2e
   b. $f_{xxz}(0,1,0) =$ 1
5. Let $f(x,y) = \sqrt{x^2+y^2}$,then the equation of the tangent plane at $P(-3,4,5)$ is 2x − 7y = 5
6. If the temperature at the point (x,y,z) is given by $T(x,y,z) = 80 + 5e^{-z}(x^{-2}+y^{-1})$,then
   a. Find the direction from the point (1,4,8) in which temperature decreases most rapidly ; 2i − 3j
   b. Find the maximum rate of temperature at the point (1,4,8): $\sqrt{41}$
7. If $z = f(x-y)$, then $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} =$ ______
8. $\int_0^1 \int_{3y}^3 e^{x^2}\,dx\,dy =$ 3/2 (e−1)
9. If $f(x,y) = e^{x+2y}$,then the total differential $df =$ ______
10. Let $xyz = \ln(x+y+z)$ then
   a. $\frac{\partial z}{\partial x} =$ $\frac{1}{x+y+z}$
   b) $\frac{\partial z}{\partial y} =$ $\frac{1}{x+y+z}$

2

...ied Maths II Final Exam

PART II: WORKOUT

SHOW ALL THE NECESSARY STEPS CLEARLY AND NEATLY

1. Use the definition of limit to verify $\lim_{(x,y)\to(1,1)}(x^2+x+y)=3$ **(4 Pts)**

$f_x = 2x$

$f_y = 1$

$D(a,b) = f_{xx}(a,b)\, f_{yy}(a,b) - (f_{xy}(a,b))^2$

2. Find the local extreme values and saddle point(s) (if any) of the function $f(x,y) = 3xy - x^2y - xy^2$ **(5 Pts)**

$f_x =$

3

3 a) Evaluate $\iint_R \ln(x^2+y^2)\,dA$,where $R=\{(x,y)/x^2+y^2\leq 1\}$ (3 Pts)

b) Find the volume of the solid under the plane $x+2y-z=0$ and above the region bounded by $y=x$ and $y=x^4$ (3 Pts)

4

4. A rocket is launched with a constant thrust corresponding to an acceleration of u ft/s^2. Ignoring air resistance, the rocket's height after t seconds is given by $f(t,u) = \frac{1}{2}(u-32)t^2$ ft .Fuel usage for t seconds is proportional to u^2t and the limited fuel capacity of the rocket satisfies the equation $u^2t = 10{,}000$. Find the value of u that maximizes the height that the rocket reaches when the fuel runs out. **(5 Pts)**

5

1. Find $\lim\limits_{(x,y)\to(e,1)} \ln \frac{x}{y}$ (3 points)

12. Let

$$f(x,y)=\begin{cases}\dfrac{x^3y^3}{x^{12}+y^4}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

Show that f is not continuous at the series $(0,0)$ (5 points)

13. Let $f(x,y)=x^4-6x^2-3xy^3+17;\ (-1,2)$ Find the

a. first partial derivatives

b. All second partial derivatives of f at the given point.

c. The gradient of $f(-1,2)$, if any

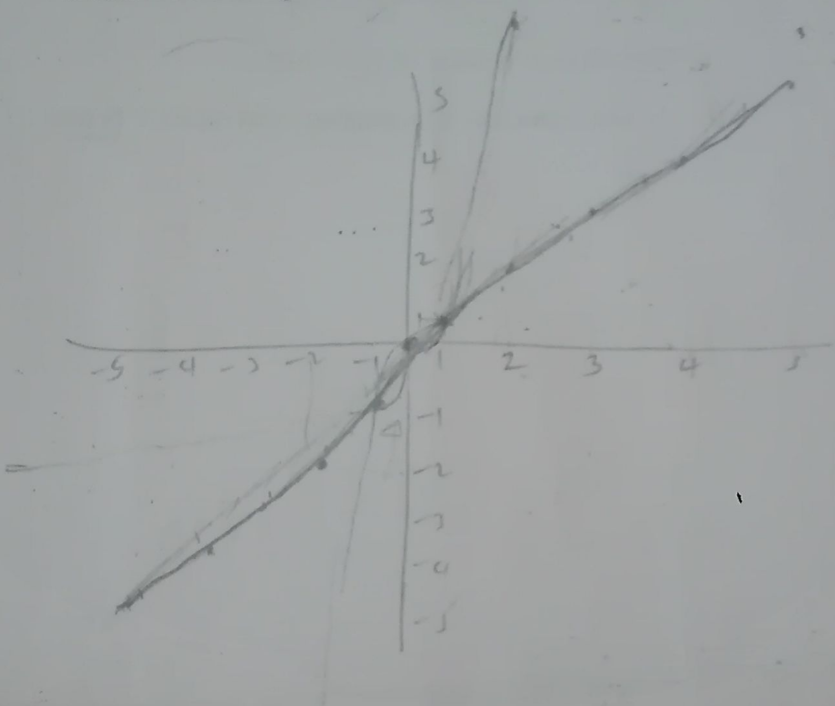
d. $D_u f(-1,2)$ Where $u = \frac{1}{\sqrt{2}} i + \frac{1}{\sqrt{2}} j$ is a unit vector,

e. The maximum value of $D_u f(-1,2)$

f. In what direction is increasing most rapidly? (9 pts)

14 Let $Z = x^2 y e^{y}$, $x = \sin t$ and $y = t^3$. Find $\frac{dz}{dt}$

17. Let R be the region between the graph of $y = x$ and $y = x^3$. Evaluate $\iint_R (x-1)dA$



x = 0, 1, -1

3. Find the area of the region R bounded by the graph of $f(x)=x^2$ and the line $x=1$, $x=3$ and the x-axis. (3 points)

$A = \int_1^3 \pi(f(x))^2 dx$

$\int_1^3 \pi (x^2)^2 dx$

$\pi \int_1^3 (x^4) dx$

$\pi \int_1^3 (x^4) dx$

$\pi \left(\frac{x^5}{5}\right) \Big|_1^3$

$\pi \left(\frac{3^5}{5}\right) - \left(\pi \frac{1^5}{5}\right)$

$\pi \frac{243}{5} - \left(\frac{\pi}{5}\right)$

$\frac{243\pi}{5} - \frac{\pi}{5} = \frac{242\pi}{5}$ sq.m

0

X

4. A radio active element has half-life of ln2 weeks. If e^3 kg present at a given time. How much will be left after 3 weeks? (3 points)

soln

$f(t) = ce^{kt}$

$f(0) = ce^{k(0)}$

$e^3 kg = ce^0$

$c = e^3$

$f(3) = ce^{k(3)}$

$\frac{\frac{e^3}{2}}{e^3} = \frac{e^3 e^{3k}}{e^3}$

$e^{3k} = \frac{e^3}{2e^3}$

$\ln e^{3k} = \frac{1}{2} \ln$

$3k = \frac{1}{2} \ln$

$k = \frac{1}{2} \ln \cdot \frac{1}{3}$

$k = \frac{1}{6} \ln$

1.5

4

5. Find the volume of the solid obtained by revolving the region between the graph of $f(x)=2x+3$ and $g(x)=x^2$ about x-axis.(3 points)

$V=\int_a^b \pi (f(x))^2-(g(x))^2 dx$

$2x+3=x^2$

$x^2-2x-3=0$

$x=-1,\ x=3$

$V=\int_{-1}^{3} \pi (2x+3)^2-(x^2)^2 dx$

$V=\int_{-1}^{3} \pi (4x^2+12x+9-x^4) dx$

$\pi \int_{-1}^{3} 4x^2+12x+9-x^4 dx$

$\pi \left(\frac{4x^3}{3}+\frac{12x^2}{2}+9x-\frac{x^5}{5}\right)\Big|_{-1}^{3}$

$\pi\left(\left(\frac{4}{3}(3)^3+\frac{12(3)^2}{2}+9(3)-\frac{3^5}{5}\right)-\left(\frac{4}{3}(-1)^3+\frac{12(-1)^2}{2}+9(-1)-\frac{(-1)^5}{5}\right)\right)$

$\pi\left(\left(36+54+27-\frac{243}{5}\right)-\left(-\frac{4}{3}+6-9+\frac{1}{5}\right)\right)$

$\pi(117-48.6)-\left(\frac{-20+3-45}{15}\right)$

$\pi(117-48.6-\left(\frac{\ }{15}\right)$

$\pi(68.4)-\left(\frac{-65+3}{15}\right)$

$\pi(68.4+12.4)=80.8\pi$ (cubic)

2.5

6. Show that (5 points)

a) $\int \sin ax \cos ax\, dx=\frac{1}{2a}\sin^2 ax+c$

b) Evaluate $\int \sin 3x \cos 4x dx$

b). $\int \sin 3x \cos 4x dx$

$\int \frac{1}{2}(\cos(3-4)x+\frac{1}{2}(\cos(3+4)x$

$\int \frac{1}{2}\cos x+\frac{1}{2}\cos 7x)$

$-\frac{1}{2}\int \cos x+\frac{1}{2}\int \cos 7x)$

$-\frac{1}{2}\sin x+\frac{1}{2}\cos 7x$

$-\frac{1}{2}\sin x+\frac{1}{14}\cos 7x$

0.5

a). $\int \sin ax \cos ax dx=\frac{1}{2}a\sin^2 ax+c$

soln $\int \sin ax \cos ax dx=\int \frac{1}{2}\cos(a-a)x+\frac{1}{2}\cos(a+a)x$

$\frac{1}{2}(\cos(0)x$

$0+\frac{1}{2}\int \cos 2ax)$

$\frac{1}{2a}\sin^2 ax+c$

5

$\cos 2a=\sin^2$

(10)

Scanned by CamScanner

ADAMA UNIVERSITY
SCHOOL OF HUMANITIES AND NATURAL SCIENCE
APPLIED MATHEMATICS II(Math.132)
FINAL EXAMINATION

Date: June 19, 2010
Time allowed: 2hrs
Total Marks: 40 points
Program: Degree Extension(winget)

NAME____________________________

ID.NO.____________________________

DEPARTEMENT____________________________

SECTION____________GROUP____________

INSTRUCTOR'S NAME____________________________

INSTRUCTION:

This examination has 10 short answers and 6 work out problems. Attempt all questions.

PART I: SHORT -ANSWER QUESTIONS.

Read carefully and give the answer in its most simplified form. (each blank space carrier 1 point)

1. Let $f(x, y) = \ell n(4 - x^2 - y^2)$. Then the (a) domain of f is _____________.

   (b) range of f is_______________.

2. $$\lim_{(x,y)\to(0,0)}\left[1 - \frac{x^2+y^2}{\tan(x^2+y^2)}\right] =$$ ______________.

3. $$\lim_{(x,y)\to(0,1)}\left[e^{x^2+x}\ell n(e\,y^2)\right] =$$ _______________________.

4. The value of for which $f(x,y)=\begin{cases}\dfrac{x^3+y^3+x^2+y^2}{x^2+y^2} & (x,y)\neq(0,0)\\ A & (x,y)=(0,0)\end{cases}$ is continuous

   at the origin is____________________.

5. If $f(x,y)=\cos(xy^2)$, then

   a) $f_x(x,y)=$______________.

   b) $f_{yx}(x,y)=$______________.

6. Let $f(x,y) = e^{x^2+y^2} + x\ln y$. Then the total differential

   d f= ______________________________.

7. The directional derivatives of $f(x,y)=e^{x^2y^3}$ at P(1,-1) in the direction toward Q(2, 3) is = ____.

8. The vector normal to the level surface $x^2+2xy-yz+3z^2=5$ at the point R(1,1 -1) is____.

9. Let $f(x,y) = x^2 - 4x + y^2 + 3y + 7$. Then the critical point(s) of f is/are ______________________.

10. The equation of tangent plane to the surface

$\sin(x+y) + \tan(y+z) = 1$ at $(\frac{\pi}{4}, \frac{\pi}{4}, -\frac{\pi}{4})$ is ______________________.

PARTII: WORK - OUT PROBLEMS.
Solve each problem in this part in detail giving the necessary justification in the space provided.

1. Use $\varepsilon - \delta$ definition of limit prove that $\lim_{(x,y)\to(0,0)} (x + y^2) = 0$. (4 points)

2. Find $\frac{\partial w}{\partial r}$ where $w = e^{2x-y+3z^2}$ and $x = r+s-t$, $y = 2r-3s$, $z = \cos(rst)$. (5 points)

3. Decide whether the limit $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$ exists or not. **(5 points)**

4.Using tangent plane approximation approximate $\sqrt{(3.02)^2+(-4.98)}$. **(4 points)**

14

5. Let $f(x,y)=3x^2-3xy^2+y^3+3y^2+4$. **(5 points)**

(a)Find all critical points.

(b)Determine whether each critical point yields a relative minimum value, a relative maximum value or saddle point.

6. A container in $\Re^3$ has the shape of the cube $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$. A plate is placed in the container in such away that it occupies the portion of the plane $x + y + z = 1$ That lies in the cubical container. If the container is heated so that the temperature at each point (x, y, z) is given by $T(x, y, z) = 3 - x^2 - y^2 - z^2$ in hundreds of degree Celsius what are the hottest and coldest points on the plate. **(5 points)**

16

Scanned by CamScanner

July 6, 2009 math 132 final exam GOOD LUCK

Part I: Blank Space (write the simplified answer on the space provided)
Each blank space worth 2 pts

1. Express the polynomial function $f(x) = x^3 - 4x^2 + 3x + 2$ as a Taylor polynomial about $x = -1$.
   Ans: ______

2. Let $f(x,y) = \sqrt{16 - x^2 - y^2}$ then
   a) domain of $f(x,y)$ = ______
   b) range of $f(x,y)$ = ______

3. $\lim\limits_{(x,y)\to(-2,\sqrt{2})} \dfrac{x^4 + x^2y^2 - 6y^4}{x^2 - 2y^2}$ = ______

4. Let $x^2 + \sin xy = -y^2$ then $\dfrac{dy}{dx}$ = ______

5. If $f(x,y) = xe^{xy}$, find all the second partial derivatives at $(1, 0)$.
   Ans: a) ______ b) ______
   c) ______ d) ______

6. Let $a = i - j$ and $b = 3i + 3j$, and $D_a f(1,2) = 6\sqrt{2}$ and $D_b f(1,2) = -2\sqrt{2}$ then
   $f_x(1,2)$ = ______ and $f_y(1,2)$ = ______

7. Let $f(x,y,z) = \ln(x^2 + y^2 + z^2)$ and $u = ai + bj + ck$ be a unit vector then
   $D_u f(2,-1,1)$ in terms of a,b and c is ______

8. The power series representation of $\int_0^x x e^{x^3} dx$ = ______

9. If $W = f(2x - 3y, 3y - 4z, 4z - 2x)$ then $6W_x + 4W_y + 3W_z$ = ______

17

July 6, 2009 math 132 final exam GOOD LUCK

Part II: Workout (show all the necessary steps clearly and neatly)

1. Find the Taylor series of $f(x) = \ln(2x)$ at $a = \frac{1}{2}$. (5pts)

Scanned by CamScanner

2. If $z = f(x, y)$ where $x = r\cos\theta$ and $y = r\sin\theta$ then show that

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$

(5pts)

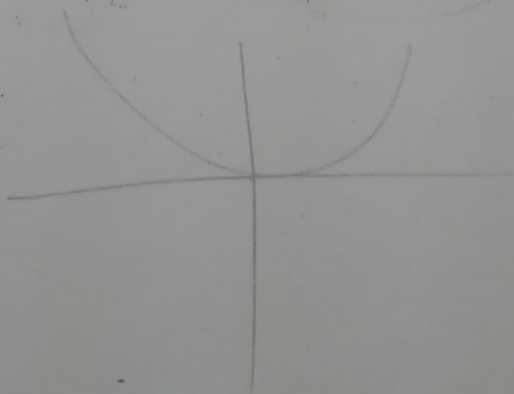
$x^2+y^2-1 >= 0$

$x^2+y^2 \geq$

$2x^2 = x^2+1$

$x^2 = 1$

$x = \pm 1$



4. *Let* $f(x, y) = x^2 - 2xy + y^3$ then find

a) all critical points of f.

b) Determine the relative extreme value(s) and saddle points of f.

c) Find the equation of the tangent plane to the graph of f at (2,-1,7).

5. The temperature T of a metal ball is inversely proportional to the distance from the centre of the ball, which we take to be the origin. The temperature at the point (1, 2, 2) is 90^0c.

a) Find the rate of change of T at (1,2,2) in the direction towards the points (2,1,3)

b) Find the maximum rate of change of T at (1,2,2)? (6pts)

Scanned by CamScanner